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PRE BOARD EXAM

TIME : 3 hrs **XII – MATHEMATICS(041)** **M.M : 80**

SECTION –A (1 MARKS each)

1. If y = sin x^o find $\frac{dy}{dx}$.

   (a) $\frac{\pi}{180}\cos x^\circ$ (b) $\frac{\pi}{180}\sin x^\circ$ (c) $\frac{180}{\pi}\cos x^\circ$ (d) $\frac{180}{\pi}\sin x^\circ$

2. The solution of $x^2\frac{d^2x}{dy^2} - x\frac{dy}{dx} + y = 2\log x\ is\ y = (c_1 + c_2\log x).x + 2g(x), then\ g(x) is$

   (a) (2 + log(x)) (b) (1 + log(x)) (c) (2 - log(x)) (d) log(x)

3. If the direction cosines of a line are K,K,K, then

   (a) K > 0 (b) 0 < K < 1 (c) K= $\pm\sqrt{3}$ (d) K= $\pm\frac{1}{\sqrt{3}}$

4. A and B are events such that P(A)=0.5, P(B)=0.4 and P$(A \cup B) = 0.1$ then P$(A^| \cup B^|)$ equals to

   (a) $\frac{2}{3}$ (b) $\frac{1}{10}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$.

5. Given matrix $A = [a_{ij}]$ of order 3 × 3 whose elements are given by $a_{ij} = \frac{(2j-i)^2}{i+j}$, write the element a_{22} of matrix *A*.

   (a) 4 (b) 1 (c) 2 (d) 0

6. If P = $\begin{bmatrix} 1 & 4 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ and Q =PP^T then value of $|Q|$ is

   (a) 2 (b) -2 (c) 1 (d) 0.

7. Value of the expression $\left|2\vec{a}\ X\ \vec{b}\right|^2 + \left(\vec{a}.2\vec{b}\right)^2$ is

   (a) $4\vec{a}.\vec{b}$ (b) 4ab (c) $4a^2b^2$ (d) $16a^4b^4$

8. The optimal value of the objective function is attained at points

   (a) Given by intersection of in equation with y-axis only
   (b) Given by intersection of in equation with x-axis only
   (c) Given by corner points of the feasible region
   (d) None of these

9. The position vector of a point which divides the joins of points with position vector $\vec{a} + \vec{b}\ and\ \overrightarrow{2a} - \vec{b}$ in the ratio 1:2 is

   (a) $\frac{3\vec{a}+2\vec{b}}{2}$ (b) $\vec{a}$ (c) $\frac{5\vec{a}-\vec{b}}{3}$ (d)$\frac{4\vec{a}+\vec{b}}{3}$

10. If x= -9 is a root of $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$, then sum of other two roots is

    (a) 11 (b) 9 (c) 6 (d) 12

11. The Principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $-\frac{\pi}{6}$.

12. if $\int \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{3}\sin^{-1} ax + C$ then value of a is

(a) 2 (b) 4 (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

13. The degree of differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + x^3 = 0$ *is* :

(a) 1 (b) 2 (c) 3 (d) not defined .

14. The number of all possible metrics of order 2X2 with each entry 0 or 1 is

(a) 8 (b) 16 (c) 4 (d) 64

15. Let A be a square matrix of order 2 x 2, A(adjA) = $\begin{vmatrix} 8 & 0 \\ 0 & 8 \end{vmatrix}$ then write the value of $|A|$.

(a) 64 (b) 8 (c) 16 (d) 0

16. Area enclosed by the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

(a) $\pi^2 ab$ (b) πab (c) $\pi a^2 b$ (d) $\pi a b^2$

17. $\int \sin x \cos x \, dx$

$(a) - \cos 2x + c$ (b) $-\sin 2x + c$ (c) $\cos 2x + c$ (d) $\frac{1}{2}\cos x + c$

18. If A = {a,b,c,d}, then a relation R= {(d,c),(c,d),(d,d)} on A is

(a) Symmetric only (b) Symmetric and transitivity only

(c) transitivity only (d) reflexive and transitivity only

ASSERTION – REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). choose the correct answer out of the following choices

(a) Both A and R are true and R is correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false
(d) R is true but A is false.

19. Assertion(A): let a relation R is defined from Set B to B such that B= {1,2,3,4} and R={(1,1),(2,2),(3,3),(1,3),(3,1)} then R is transitive.

Reason(R): $A\ relation\ R\ in$ set A is called transitive, if (a,b) ∈ R and (b,c) ∈ $R \Rightarrow (a,c) \in R$, ∀ a,b,c ∈ R.

20. Assertion(A) : Maximize value of z=3x+2y, subject to constraint: x+y ≤ 2, $x \geq 0, y \geq 0$ will be obtained at point (2,0).

Reason(R): In a bounded feasible region, it always exist a maximum and minimum value.

SECTION –B(2 marks each)

21. Find the value of $\sin\left[\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$.

22. The x coordinate of a point on the line joining the points P(2,1,1) and Q (3,1,-2) is 5. Find Y – coordinate .

OR

If the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ & $\frac{x-1}{3k}=\frac{y-2}{1}=\frac{z-3}{-5}$ are perpendicular then find k.

23. Find the absolute maximum and absolute minimum value of the function f(x)=4x $-\frac{x^2}{2}$, $x\in\left[-2,\frac{9}{2}\right]$.

24. Differentiate w.r.t x : $\tan^{-1}\left(\frac{a\cos x-b\sin}{b\cos x+a\sin x}\right)$.

25. Find the angle between the vector $\vec{a}$ & $\vec{b}$ if $|\vec{a}|=2,|\vec{b}|=6$ and $\vec{a}\ X\ \vec{b}=\hat{\imath}+2\hat{\jmath}+6\hat{k}$.

SECTION C(3 marks each)

26. $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$

27. $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ OR $\int_0^{\pi/4}\frac{(\sin x+\cos x)}{9+16\sin 2x}dx$.

28. Solve: $x\,dy-y\,dx=\sqrt{x^2+y^2}dx$

OR

Find particular solution for given differential equation satisfying the given conditions $\frac{dy}{dx}+y\cot x=4x\,cosec\,,y=0\ \&\ x=\frac{\pi}{2}$.

29. $\int x\tan^{-1}x\ dx$.

30. Minimize and Maximize z=3x+9y subject to constraint: x+3y$\leq 60, x+y\geq 10,\ x\leq y,\ x\geq 0, y\geq 0$.

31. $\int\frac{dx}{x^{1/3}+x^{1/6}}$.

SECTION D(5 marks each)

32. Show that each of the relation R in set A= { x ∈ z: $0\leq x\leq 12$} given by R= {(a,b):$|a-b|$ is a multiple of 4.

OR

Let A = R – {3} and B = R – {1}. Consider the function $f:A\to B$ defined by f(x) =$\frac{x-2}{x-3}$ prove that f is bijective.

33. Find the area of the region lying in the first quadrant and enclosed by the x-axis, the line y=x and the circle $x^2+y^2=32$.

34. Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ & $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

OR

Show that the lines $\frac{x-3}{1}=\frac{y-3}{2}=\frac{z+4}{2}$ & $\frac{x-5}{3}=\frac{y+2}{2}=\frac{z}{6}$ are intersecting. Also find point of intersection.

35. Solve by matrix method $\frac{1}{x}-\frac{3}{y}+\frac{3}{z}=10,\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10\ and\ \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$.

SECTION E(case study)(4 marks each)

36. Read the following passage and answer the questions given below.

In a elliptical sport field the authority wants to design a rectangular soccer field with maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.

(iii) Use First Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

37. In a college, an architecture designs an auditorium for its cultural activities purpose. The shape of the floor of the auditorium is rectangular and it has a fixed perimeter, say P. Based on the above information, answer the following questions.

(i) If l and b represents the length and breadth of the rectangular region, then find the relationship between l,b,P.

(ii) Find the area(A) of the floor, as a function of l.

(iii) College manager is interested in maximizing the area of the floor A. for this purpose, find the value of l

OR

Find the maximum area of the floor.

38. If f(x) is a continuous function defined on [a,b] then $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx.$

On the basis of the above information, answer the following equations Evaluate:

(1) Evaluate: $\int_1^2 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{3-x}} dx$ (ii) Evaluate: $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin(x)}dx}{\sqrt{\sin(x)}+\sqrt{\cos(x)}}$.

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